BRIN MATHEMATICS RESEARCH CENTER

Workshop: Branching Processes and Reaction Diffusion Equations

MARCH 8 - 10, 2023

Speakers

Yuri Bakhtin (New York University)
Henri Berestycki (CNRS / EHESS)
Julien Berestycki (University of Oxford)
Cole Graham (Brown University)
Pratima Hebbar (Grinnell College)
Christopher Henderson (University of Arizona)
Gautam Iyer (Carnegie Mellon University)
Elena Kosygina (Baruch College)
Jim Nolen (Duke University)
Alexei Novikov (Penn State University)
Sarah Penington (University of Bath)
Jack Xin (UC Irvine)
Andrej Zlatos (UC San Diego)

Organizers

Leonid Koralov, University of Maryland
Lenya Ryzhik, Stanford University

About the Workshop

Branching processes and reaction-diffusion equations are mathematical tools used to model the evolution of particle systems, with applications in biology, chemistry and other sciences. The goal of this workshop is to bring together some of the leading experts in these and related fields in probability and partial differential equations, to present their recent research and exchange ideas, and to provide an opportunity for graduate students and young researchers to develop their knowledge of these fields.

BRINMRC.UMD.EDU

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DEPARTMENT OF MATHEMATICS

Contents

Workshop Overview

Branching processes and reaction-diffusion equations are mathematical tools used to model the evolution of particle systems, with applications in biology, chemistry and other sciences. The goal of this workshop is to bring together some of the leading experts in these and related fields in probability and partial differential equations, to present their recent research and exchange ideas, and to provide an opportunity for graduate students and young researchers to develop their knowledge of these fields.

Organizing committee

Leonid Koralov, University of Maryland

Lenya Ryzhik, Stanford University

Schedule at a Glance

	Wednesday	Thursday	Friday
9:00	Breakfast	Breakfast	Breakfast
10:00	Gautam Iyer	Alexei Novikov	Jim Nolen
11:00	Christopher Henderson	Pratima Hebbar	Henri Berestycki
12:00	Coffee break	Coffee break	Coffee break
13:00	Sarah Penington	Cole Graham	Yuri Bakhtin
14:00	Lunch	Lunch	Louigi Addario-Berry
15:00	Andrej Zlatos	Julien Berestycki	Lunch
16:00	Coffee break		
17:00	Jing An	Sergiu Klainerman	
	Jack Xin	High tea	

Workshop Schedule

Wednesday, March 8, 2023

9:00 - 9:25 Breakfast

9:25 - 9:30 Doron Levy (University of Maryland/Director, Brin MRC)
Opening

9:30 - 10:15 Gautam Iyer (Carnegie Mellon University)
Using mixing to speed up diffusion

10:30 - 11:15 Christopher Henderson (University of Arizona)
The shape defect function and stability of traveling waves

11:15 - 11:45 Coffee break

11:45 - 12:30 Sarah Penington (University of Bath)
Branching Brownian motion with mean-dependent branching

12:30 - 2:00 Lunch

2:00 - 2:45 Andrej Zlatos (UC San Diego)
Homogenization in front propagation models

2:45 - 3:15 Coffee break

3:15 - 4:00 Jing An (Duke University)
On the pushed-to-pulled front transitions in (advection)-reaction-diffusion models

4:15 - 5:00 Jack Xin (UC Irvine)
Particle and Game Theoretic Methods for Front Speeds in Fluid Flows

Thursday, March 9, 2023

9:00 - 9:30 Breakfast

9:30 - 10:15 Alexei Novikov (Penn State University)
Speed-up of Langevin dynamics by mixing flows

10:30 - 11:15 Pratima Hebbar (Grinnell College)
Branching Diffusion in Periodic Media

11:15 - 11:45 Coffee break

11:45 - 12:30 Cole Graham (Brown University)
Uniqueness of KPP steady states in general domains

12:30 - 2:00 Lunch

2:00 - 2:45 Julien Berestycki (University of Oxford)
The F-KPP equation in the half-plane

3:15 - 4:15 Sergiu Klainerman (Princeton University)
Distinguished Lecture in Geometric Analysis (Kirwan 3206): Are Black Holes Real? A Mathematical Approach to an Astrophyical Question

4:15 - 4:45 High tea

Friday, March 10, 2023

9:00 - 9:30 Breakfast

9:30 - 10:15 Jim Nolen (Duke University)
Optimal growth transport

10:30 - 11:15 Henri Berestycki (CNRS / EHESS)
Uniqueness of stationary solutions of reaction-diffusion equations in general domains

11:15 - 11:45 Coffee break

11:45 - 12:30 Yuri Bakhtin (Courant Institute, NYU)
Rare transitions in noisy heteroclinic networks.

12:45 - 1:30 Louigi Addario-Berry (McGill University)
Height bounds for random trees.

1:30 - 2:00 Lunch

2:00 - 2:05 Workshop Closing

Abstracts of talks

Using mixing to speed up diffusion

Gautam Iyer

Carnegie Mellon University

Wednesday, March 8, 2023 @ 9:30 AM

Consider a (discrete time) random walk on the torus, intertwined with the action of a Lebesgue measure preserving dynamical system ϕ. What is the mixing time? The mixing time of a random walk with step size ϵ is $O(1/\epsilon^2)$. If the dynamics of ϕ is sufficiently mixing, this can be reduced to $O(\log \epsilon)$. I will talk about one discrete time example (where ϕ is Bernoulli), and one continuous time example (cellular flows), and possibly other applications (e.g. sampling).

The shape defect function and stability of traveling waves

Christopher Henderson

University of Arizona

Wednesday, March 8, 2023 @ 10:30 AM

In their original paper, Kolmogorov, Petrovsky, and Piskunov demonstrated stability of the minimal speed traveling wave with an ingenious argument based on, roughly, the decreasing steepness of the profile. This proof is extremely flexible, yet entirely not quantitative being based on compactness. On the other hand, more modern PDE proofs of the stability of traveling waves solutions to reaction-diffusion equations are highly tailored to the particular equation, fairly complicated, and often not sharp in terms of the rate of convergence. In this talk, I will introduce a natural quantity, the shape defect function, that allows a simple approach to quantifying convergence to the traveling wave for a large class of reaction-diffusion equations, including both pushed, pulled, and pushmi-pullyu equations. This is a joint work with Jing An and Lenya Ryzhik.

Branching Brownian motion with mean-dependent branching

Sarah Penington

University of Bath

Wednesday, March 8, 2023 @ 11:45 AM

We consider a continuous-space analogue of a population model introduced by Yu, Etheridge and Cuthbertson. We prove a hydrodynamic limit result that allows us to show that for a large total population size, at large times the empirical distribution of the particle positions evolves approximately according to an accelerating Gaussian wave. Based on joint work with Erin Beckman.

Homogenization in front propagation models

Andrej Zlatos

UC San Diego

Wednesday, March 8, 2023 @ 2:00 PM

Homogenization is a general principle that the dynamics of physical processes occurring in periodic or random environments often become effectively homogeneous on large space time scales. I will present results showing that homogenization occurs for reaction-diffusion equations with time-periodic spatially random as well as space-time-random KPP reactions and coefficients. These results rely on two crucial new tools: virtual linearity of KPP reaction-diffusion dynamics and a non-autonomous versions of Kingmanis subadditive ergodic theorem.

On the pushed-to-pulled front transitions in (advection)-reaction-diffusion models

Jing An

Duke University

Wednesday, March 8, 2023 @ 3:15 PM

We will discuss pushed-to-pulled transitions in front propagation that occur in various (advection)-reaction-diffusion models. Our focus is on the large-time behavior of reactive conservation laws and semi-Fisher-KPP equations, particularly in the critical case where these two equation types can be connected. We refer to this boundary as the "pushmi-pullyu" boundary. To determine a precise front asymptotic of this boundary, we employ a new weighted Hopf-Cole transform and a relative entropy approach. This talk is based on joint works with Chris Henderson and Lenya Ryzhik.

Particle and Game Theoretic Methods for Front Speeds in Fluid Flows

Jack Xin

UC Irvine

Wednesday, March 8, 2023 @ 4:15 PM

We study and compute large time front speeds of Fisher-Kolmogorov-Petrovsky-Piskunov equation in fluid flows with chaotic and stochastic streamlines based on a normalized Feynman-Kac representation and the associated genetic interacting particle (IP) method. Examples of such flows are Arnold-Beltrami-Childress (ABC) flows and their random perturbations. The method is mesh-free and self-adaptive, providing training data for efficient deep learning of the invariant measure of IP evolution in the small molecular diffusivity regime. We analyze a curvature dependent level set Hamilton-Jacobi equation arising in turbulent combustion, and show the existence of effective front speeds in cellular (BC) flow. To overcome non-coercivity and non-convexity of the Hamiltonian, we leverage Kohn-Serfaty deterministic game characterization and streamline structure of the flow.

Speed-up of Langevin dynamics by mixing flows

Alexei Novikov

Penn State University

Thursday, March 9, 2023 @ 9:30 AM

I discuss how to access and speed up the convergence towards stationary Gibbs measures by adding flows with varying mixing, dynamic and geometric properties.

Branching Diffusion in Periodic Media

Pratima Hebbar

Grinnell College

Thursday, March 9, 2023 @ 10:30 AM

We describe the behavior of branching diffusion processes in periodic media. For a super-critical branching process, we distinguish two types of behavior for the normalized number of particles in a bounded domain, depending on the distance of the domain from the region where the bulk of the particles is located. At distances that grow linearly in time, we observe intermittency (i.e., the k-th moment dominates the k-th power of the first moment for some k), while, at distances that grow sub-linearly in time, we show that all the moments converge. A key ingredient in our analysis is a sharp estimate of the transition kernel for the branching process, valid up to linear in time distances from the location of the initial particle.

Uniqueness of KPP steady states in general domains

Cole Graham

Brown University

Thursday, March 9, 2023 @ 11:45 AM

Reaction–diffusion equations on Dirichlet domains model populations in territory with hostile boundary. Steady states represent equilibria between reproduction in the interior and mortality at the boundary. On unbounded domains, such steady states are poorly understood even for simple KPP reactions. In this talk, I will show that KPP steady states are unique on domains that satisfy a certain spectral nondegeneracy condition. This is joint work with Henri Berestycki.

The F-KPP equation in the half-plane

Julien Berestycki

University of Oxford

Thursday, March 9, 2023 @ 2:00 PM

It has been shown by H. Berestycki and G. Cole (2022) that the F-KPP equation $\partial_t u = \frac{1}{2}\Delta u + u(1-u)$ in the half-plane with Dirichlet boundary conditions admits travelling wave solutions for all speeds $c \geq c^* = \sqrt{2}$.

We show that the minimal speed travelling wave Φ is in fact unique (up to shift) and give a probabilistic representation as the Laplace transform of a certain martingale limit associated to the branching Brownian motion with absorption. This representation allows us to study the asymptotic behaviour of Φ away from the boundary of the domain, proving that

$$\lim_{y\to\infty} \Phi\left(x + \tfrac{1}{\sqrt{2}}\log y, y\right) = w(x)$$

where w is the usual one-dimensional critical travelling wave.

We are able to extend our result to the case of the half-space $\mathbb{H}^d = \{x \in R^d : x_1 \geq 0\}$. Finally, if time allows, I will also mention some results regarding the convergence towards the critical travelling wave.

This is based on joint work with Graham Cole, Yujin Kim and Bastien Mallein.

Distinguished Lecture in Geometric Analysis (Kirwan 3206): Are Black Holes Real? A Mathematical Approach to an Astrophyical Question

Sergiu Klainerman

Princeton University

Thursday, March 9, 2023 @ 3:15 PM

The question whether black holes are real can be approached mathematically by addressing basic issues concerning their rigidity, stability and how they form in the first place. I will review these and focus on recent results concerning the nonlinear stability of slowly rotating Kerr black holes.

Optimal growth transport

Jim Nolen

Duke University

Friday, March 10, 2023 @ 9:30 AM

Unbalanced optimal transport between two positive measures is a recent generalization of optimal transport to case where the initial and target measures have unequal mass – there have been several works on this topic in the last decade. We consider a constrained version of unbalanced transport where mass can only grow (perhaps modeling a growing biological tissue). This leads to the definition of a quasi-metric on space of positive measures. I will discuss some novel features of this framework and related open issues. This is ongoing work with Yuqing Dai.

Uniqueness of stationary solutions of reaction-diffusion equations in general domains

Henri Berestycki

CNRS / EHESS

Friday, March 10, 2023 @ 10:30 AM

When considering propagation for reaction-diffusion equations in general domains, one is led to study stationary solutions of semi-linear elliptic equations with various boundary conditions. This talk is about the question of uniqueness of positive solutions for such problems with positive reaction terms. We first show that uniqueness does not hold in general. Then, I will present various general geometrical conditions that yield uniqueness. In the process, we uncover an unexpected wealth of phenomena. I report in this talk on joint work with Cole Graham.

Rare transitions in noisy heteroclinic networks.

Yuri Bakhtin

Courant Institute, NYU

Friday, March 10, 2023 @ 11:45 AM

We study white noise perturbations of planar dynamical systems withheteroclinic networks in the limit of vanishing noise. We show thatthe probabilities of transitions between various cells that thenetwork tessellates the plane into decay as powers of the noisemagnitude, and we describe the underlying mechanism. A metastabilitypicture emerges, with a hierarchy of time scales and clusters ofaccessibility, similar to the classical Freidlin-Wentzell picture butwith shorter transition times. We discuss applications of our resultsto homogenization problems and to the invariant distributionasymptotics. At the core of our results are local limit theorems forexit distributions obtained via methods of Malliavin calculus. Jointwork with Hong-Bin Chen and Zsolt Pajor-Gyulai.

Height bounds for random trees.

Louigi Addario-Berry

McGill University

Friday, March 10, 2023 @ 12:45 PM

I will explain how an old tool (the Foata-Fuchs bijection) can be used to settle a family of conjectures about the heights of random trees in several different random tree models, including family trees of branching processes, simply generated trees, and random trees with given degree sequences. This is based on joint work with Serte Donderwinkel.

The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

List of Participants

Alexei Novikov, Penn State University
Andrej Zlatos, UC San Diego
Arturo Arellano, McGill University
Christopher Henderson, University of Arizona
Cole Graham, Brown University
Doron Levy, University of Maryland/Director, Brin MRC
Douglas Dow, NYU
Elena Kosygina, Baruch College and the CUNY Graduate Center
Francois Hamel, Universite d'Aix-Marseille
Gautam Iyer, Carnegie Mellon University
Henri Berestycki, CNRS / EHESS
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Jongwon Kim, UC Irvine
Julien Berestycki, University of Oxford
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Lenya Ryzhik, Stanford University
Louigi Addario-Berry, McGill University
Maria Delyianni , University of Arizona
Pratima Hebbar, Grinnell College
Ryan Patterson, University of Arizona
Sarah Penington, University of Bath
Sergiu Klainerman, Princeton University
Yujin Kim, NYU
Yuri Bakhtin, Courant Institute, NYU