

Advances in Higgs Bundles

April 15 - 19, 2024

About the Workshop

The goal of the workshop will be to bring together experts working on topics related to moduli spaces of Higgs bundles. These include mirror symmetry, the geometric Langlands program, and relations with special representation varieties.

Organizers

Steve Bradlow, UIUC Bill Goldman, University of Maryland Richard Wentworth, University of Maryland

Speakers

Joergen Andersen, University of Southern Denmark Brian Collier, University of California-Riverside Ron Donagi, University of Pennsylvania Laura Fredrickson, University of Oregon Oscar Garcia-Prada, ICMAT Peter Gothen, University of Porto Andriy Haydys, Free University of Brussels Sigi He, Morningside Center Jochen Heinloth, University of Duisberg Nigel Hitchin, University of Oxford Ludmil Katzarkov, University of Miami Marina Logares, Complutense University Rafe Mazzeo, Stanford University Andy Neitzke, Yale University Andre Oliveira, University of Porto Ana Peon-Nieto, University of Birmingham Nathaniel Sagman, University of Luxembourg Laura Schaposnik, University of Illinois at Chicago Peter Smillie, University of Heidelberg Harmut Weiss, University of Kiel Anna Wienhard, MPIM Mike Wolf, Georgia Tech University

brinmrc.umd.edu



CSIC Building, 4th Floor 8169 Paint Branch Drive University of Maryland College Park, MD 20742



DEPARTMENT OF MATHEMATICS

Contents

Workshop Overview 5					
Workshop Schedule 6					
Abstracts of talks					
Nigel Hitchin					
Peter Gothen					
Oscar Garcia-Prada					
Andy Neitzke					
Ron Donagi					
Andriy Haydys					
Panos Dimakis					
Rafe Mazzeo					
Mike Wolf					
Sebastian Schultz					
Nathaniel Sagman					
Jochen Heinloth					
Siqi He					
Charles Ouyang					
Johannes Horn					
Hartmut Weiss					
Brian Collier					
Peter Smillie					
John Loftin					
Andre Oliveira					
Steven Rayan					
The Brin Mathematics Research Center 21					
List of Participants 22					

Schedule at a Glance

8.00	Monday	Tuesday	Wednesday	Thursday	Friday
0.00	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
9:00	Hitchin	Haydys	Sagman	Ouyang	Loftin
10:00					
11:00	Gothen	Dimakis	Heinloth	Horn	Oliveira
	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
12:00	Garcia-Prada	Mazzeo	Не	Weiss	Rayan
13:00	Lunch	Lunch	Lunch on your own	Lunch	Lunch on your own
14:00					
15:00	Neitzke	Wolf		Collier	
16.00	Coffee Break	Coffee Break		Coffee Break	
17.00	Donagi	Schultz		Smillie	
17:00	High Tea				
18:00		1		1	1

Workshop Overview

The goal of the workshop will be to bring together experts working on topics related to moduli spaces of Higgs bundles. These include mirror symmetry, the geometric Langlands program, and relations with special representation varieties.

Organizing committee

STEVE BRADLOW, UIUC

BILL GOLDMAN, University of Maryland

RICHARD WENTWORTH, University of Maryland

Workshop Schedule

Monday, April 15, 2024

8:00 - 8:50	Breakfast
8:50 - 9:00	DORON LEVY (University of Maryland/Director, Brin MRC) $Opening$
9:00 - 10:00	NIGEL HITCHIN (University of Oxford) SU-infinity Higgs bundles: an update
10:15 - 11:15	PETER GOTHEN (University of Porto) The Conformal Limit and Projective Structures
11:15 - 11:45	Coffee Break
11:45 - 12:45	OSCAR GARCIA-PRADA (ICMAT, Madrid) Towards a Counting of Non-Maximal Toledo Components
12:45 - 2:30	Lunch
2:30 - 3:30	ANDY NEITZKE (Yale University) Higgs bundles, exact WKB and dual Poisson-Lie groups
3:30 - 4:00	Coffee Break
4:00 - 5:00	RON DONAGI (University of Pennsylvania) On the geometry of meromorphic Higgs bundles
5:00 - 6:00	High Tea

TUESDAY, APRIL 16, 2024

8:00 - 9:00	Breakfast
0.00 0.00	DICLIMINIOI

- 9:00 10:00 ANDRIY HAYDYS (Free University of Brussels) Z_2 harmonic forms and spinors, I
- 10:15 11:15 PANOS DIMAKIS (Universite du Quebec) On a conjecture of Simpson
- 11:15 11:45 Coffee Break
- 11:45 12:45 RAFE MAZZEO (Stanford University) Z_2 harmonic forms and spinors, II
- 12:45 2:30 Lunch
- 2:30 3:30 MIKE WOLF (Georgia Tech University) Rays of Holomorphic Differentials
- 3:30 4:00 Coffee Break
- 4:00 5:00 SEBASTIAN SCHULTZ (Johns Hopkins University) G2 Spectral Networks

WEDNESDAY, APRIL 17, 2024

8:00 - 9:00 Breakfast

- 9:00 10:00 NATHANIEL SAGMAN (University of Luxembourg) Complex Harmonic Bundles and Extending Bers' Theorem
- 10:15 11:15 JOCHEN HEINLOTH (University of Duisberg-Essen) Geometry of G-Hodge bundles on curves
- 11:15 11:45 Coffee Break
- 11:45 12:45 SIQI HE (Morningside Center, Beijing) Hitchin Morphism for Projective Variety
- 12:45 2:30 LUNCH ON YOUR OWN
- 7:00 9:00 Conference Dinner

THURSDAY, APRIL 18, 2024

- 8:00 9:00 Breakfast
- 9:00 10:00 CHARLES OUYANG (Washington University) Higgs bundles and SYZ geometry
- 10:15 11:15 JOHANNES HORN (Goethe Universitat Frankfurt) Visible Lagrangians for Hitchin systems
- 11:15 11:45 Coffee Break
- 11:45 12:45 HARTMUT WEISS (University of Kiel) Singular Solutions to Hitchin's Equation and Harmonic Maps to the Conformal 3-Sphere
- 12:45 2:30 Цилсн
- 2:30 3:30 BRIAN COLLIER (University of California-Riverside) Slodowy Slices, Variations of Hodge Structures and Anosov Representations
- 3:30 4:00 Coffee Break
- 4:00 5:00 PETER SMILLIE (University of Heidelberg) High Energy Estimates and the Labourie Conjecture

FRIDAY, APRIL 19, 2024

8:00 -	9:00	Breakfast
0.00	5.00	DIGLININGI

- 9:00 10:00 JOHN LOFTIN (Rutgers University-Newark) Cubic Differentials and Harmonic Maps into a Real Building
- 10:15 11:15 ANDRE OLIVEIRA (University of Porto) Higgs Bundles and Lie Algebroid Connections
- 11:15 11:45 Coffee Break
- 11:45 12:45 STEVEN RAYAN (University of Saskatchewan) Quantum Matter, Hyperbolic Band Structures, and Moduli Spaces
- 12:45 1:00 Workshop Closing
- 1:00 2:30 LUNCH ON YOUR OWN

Abstracts of talks

SU-infinity Higgs bundles: an update

NIGEL HITCHIN

University of Oxford

Monday, April 15, 2024 @ 9:00 AM

Nine years ago, together with Olivier Biquard, I introduced an approach to an infinite-dimensional higher Teichmueller space by replacing the compact gauge group of a Higgs bundle with the group of symplectic diffeomorphisms of the 2-sphere. The talk will discuss various issues associated with the project, and its links to other areas.

The Conformal Limit and Projective Structures

Peter Gothen

University of Porto

Monday, April 15, 2024 @ 10:15 AM

The non-abelian Hodge correspondence maps a polystable SL(2, R)-Higgs bundle on a compact Riemann surface X of genus g > 1 to a connection which, in some cases, is the holonomy of a branched hyperbolic structure. On the other hand, Gaiotto's conformal limit maps the same bundle to a partial oper, i.e., to a connection whose holonomy is that of a branched complex projective structure compatible with X. I shall explain how these are both instances of the same phenomenon: the family of connections appearing in the conformal limit can be understood as a family of complex projective structures, deforming the hyperbolic ones into the ones compatible with X. Moreover, when the Higgs bundle has zero Toledo invariant, this deformation is optimal, inducing a geodesic in the Teichmüller metric.

This is joint work with Pedro M. Silva.

Towards a Counting of Non-Maximal Toledo Components

OSCAR GARCIA-PRADA

ICMAT, Madrid

Monday, April 15, 2024 @ 11:45 AM

The well-known Milnor-Wood inequality gives a bound on the Toledo invariant of a representation of the fundamental group of a compact surface in a group of Hermitian type. While a lot is known regarding the counting of maximal Toledo components, and their role in higher Teichmüller theory, the non-maximal case remains elusive. In this talk, I will present a strategy to count the number of such non-maximal Toledo connected components. This is joint work in progress with Brian Collier and Jochen Heinloth, building on previous joint work with Olivier Biquard, Brian Collier and Domingo Toledo.

Higgs bundles, exact WKB and dual Poisson-Lie groups

ANDY NEITZKE

Yale University

Monday, April 15, 2024 @ 2:30 PM

It was pointed out by Boalch that the dual Poisson-Lie group to a Lie group G can be described as a certain moduli space of framed connections on the Riemann sphere with both regular and irregular singularities. Such moduli spaces are expected to have many interesting geometric structures, e.g. hyperkahler structures, cluster structures and associated Donaldson-Thomas invariants, related to their realizations as moduli spaces of Higgs bundles and as Coulomb branches of supersymmetric quantum field theories. These structures can be studied via the method of exact WKB and spectral networks. I will discuss how this method plays out in this case, focusing on the cases G = GL(2) and G = GL(3) where our understanding is most complete. This is a report of joint work with Anton Alexeev, Xiaomeng Xu and Yan Zhou. I will also briefly discuss the quantization, which involves the corresponding quantum groups.

On the geometry of meromorphic Higgs bundles

RON DONAGI

University of Pennsylvania

Monday, April 15, 2024 @ 4:00 PM

We explore both local and global aspects of the geometry of meromorphic Higgs moduli space and its Hitchin map. Work with Balasubramanian, Distler, Herrero and Perez.

Z_2 harmonic forms and spinors, I

ANDRIY HAYDYS

Free University of Brussels

Tuesday, April 16, 2024 @ 9:00 AM

Just as in the case of Hitchin's equations on Riemann surfaces, the corresponding moduli spaces associated with higher dimensional manifolds are non-compact in general and the non-compact ends of such moduli spaces are modelled on certain Z2 harmonic forms/spinors. I shall describe certain properties of these objects focusing on the properties of their branching sets.

On a conjecture of Simpson

PANOS DIMAKIS

Universite du Quebec

Tuesday, April 16, 2024 @ 10:15 AM

From the work of Simpson it holds that the De Rham moduli space admits a stratification with each stratum being a smooth lagrangian foliation. Simpson conjectured that the leaves of the foliation are closed in the topology of the ambient space. Based on previous work of Shulz, we give a complete proof of the conjecture when the underlying group is $SL(2, \mathbb{C})$.

Z_2 harmonic forms and spinors, II

RAFE MAZZEO

Stanford University

Tuesday, April 16, 2024 @ 11:45 AM

Picking up from Haydys' talk, I will discuss a number of developments regarding Z_2 harmonic 1-forms and Z_2 harmonic spinors, primarily on 3-manifolds. This will include some discussion of deformation theory, due to Takahashi and later Parker, and work in progress of myself and He, and the index theorem for this deformation theory, particularly when the branching set is a smoothly embedded graph, recent work of myself with Haydys and Takahashi.

Rays of Holomorphic Differentials

MIKE WOLF

Georgia Tech University

Tuesday, April 16, 2024 @ 2:30 PM

We study the geometry of rays of holomorphic Hitchin differentials in low rank. Mostly, we restrict ourselves to the classical case of $PSL(2, \mathbb{R})$, where we can depict the rays (and a dual version) as interpolating between Teichmüller and particular Thurston geodesics; there are new consequences for Thurston metric geometry. We describe limits in rank two.

G2 Spectral Networks

SEBASTIAN SCHULTZ

Johns Hopkins University

Tuesday, April 16, 2024 @ 4:00 PM

Spectral Networks are Intimately Linked to Non-Abelianization of Flat Connections, Fock-Goncharov cluster coordinates etc. I will describe this is in detail for the group G2, and will give as an application a concrete parametrization of the Nonabelian Hodge Correspondence for the Hitchin component (of the split real form of G2). This is joint work with Andy Neitzke.

Complex Harmonic Bundles and Extending Bers' Theorem

NATHANIEL SAGMAN

University of Luxembourg

Wednesday, April 17, 2024 @ 9:00 AM

Given a closed Riemann surface S of genus at least 2, Bers simultaneous uniformization theorem provides a biholomorphism between the space of quasi-Fuchsian 3-manifolds homeomorphic to SxR and the product of the Teichmuller space of S with the Teichmüller space of the conjugate of S. If we view Teichmuller space as a connected component of the PSL(2,R) character variety, then it embeds into the space of Hitchin representations of $\pi_1(S)$ into PSL(3,R), a so-called higher Teichmuller space. Similarly, the space of quasi-Fuchsian 3-manifolds, which can be seen as a subset of the PSL(2,C) character variety, embeds into the space of quasi-Hitchin representations of $\pi_1(S)$ into PSL(3,C).

Forty years after Bers theorem, Labourie and Loftin proved, using Higgs bundles and affine spheres, that the space of Hitchin representations into PSL(3,R) admits a natural complex structure. In light of this result, one could ask, hopefully and perhaps optimistically, to what extent might the Bers' theorem generalize for quasi-Hitchin representations? In this talk we introduce new objects called complex harmonic bundles and complex affine spheres, and we use them to see what we can say about extending Bers' theorem. This is work in progress, joint with Christian El Emam.

Geometry of G-Hodge bundles on curves

JOCHEN HEINLOTH

University of Duisberg-Essen

Wednesday, April 17, 2024 @ 10:15 AM

In joint work with Brian Collier and Oscar Garcia-Prada we found an algebraic interpretation of the particular stability conditions for G-Hodge bundles introduced by Biquard, Collier, Garcia-Prada and Toledo in terms of theta-stability. After recalling this description I will try to explain how we can use this interpretation to understand the geometry of the corresponding moduli spaces through wall-crossing.

Hitchin Morphism for Projective Variety

SIQI HE

Morningside Center, Beijing

Wednesday, April 17, 2024 @ 11:45 AM

The Hitchin morphism is a map from the moduli space of Higgs bundles to the Hitchin base, which is generally not surjective when the dimension of the variety is greater than one. Chen-Ngo introduced the concept of the spectral base, which is a closed subscheme of the Hitchin base. They conjectured that the Hitchin morphism is surjective to the spectral base and also proved that the surjectivity is equivalent to the existence of finite Cohen-Macaulayfications of the spectral varieties. For rank two Higgs bundles, we will discuss an explicit construction of the Cohen-Macaulayfication of the spectral variety. In addition, we will discuss several applications using the spectral base to the topology of projective variety. This talk is based on some collaborative work with J. Liu and N. Mok.

Higgs bundles and SYZ geometry

CHARLES OUYANG

Washington University

Thursday, April 18, 2024 @ 9:00 AM

Special Lagrangian 3-torus fibrations over a 3-dimensional base play an important role in mirror symmetry and the SYZ conjecture. In this talk, we discuss the construction, via Higgs bundles, of an infinite family of semi-flat Calabi-Yau metrics on special Lagrangian torus bundles over an open ball in R^3 with a Y-vertex deleted. This is joint work with S. Heller and F. Pedit.

Visible Lagrangians for Hitchin systems

JOHANNES HORN

Goethe Universitat Frankfurt

Thursday, April 18, 2024 @ 10:15 AM

We will present two examples of complex Lagrangians of Higgs bundle moduli spaces that are visible in the sense that the restriction of the Hitchin map factors through a proper subset of the Hitchin base.

Firstly, certain complex Lagrangians that appear for special Riemann surfaces called pillowcase covers. They are minimal in the sense that they factor through a 1-dimensional subset of the Hitchin base (jt. with J. Schwab).

The second class of examples is completely contained in the locus of singular fibers of the Hitchin map an exploits the geometry of these fibers (jt. with E. Franco, R. Hanson, A. Oliveira).

In last part of the talk, we will explain how a Fourier-Mukai transform relates these complex Lagrangians to embeddings of Hausel's toy model in the first case and subintegrable systems in the singular locus studied by N. Hitchin in the second.

Singular Solutions to Hitchin's Equation and Harmonic Maps to the Conformal 3-Sphere

HARTMUT WEISS

University of Kiel

Thursday, April 18, 2024 @ $11{:}45~\mathrm{AM}$

I will report on ongoing work with Sebastian Heller and Lothar Schiemanowski on solutions to Hitchin's equation which are singular along a system of simple closed curves on the underlying Riemann surface. These are constructed by gluing methods. We use them to construct harmonic maps to the conformal 3-sphere with a specified behaviour when passing through the sphere at infinity. Earlier examples of such were obtained by Heller and Heller.

Slodowy Slices, Variations of Hodge Structures and Anosov Representations

BRIAN COLLIER

University of California-Riverside

Thursday, April 18, 2024 @ 2:30 PM

Slodowy slices in the moduli space of Higgs bundles are a generalization of the Hitchin section. They are labeled by conjugacy classes of nilpotent elements in a complex simple Lie algebra. For special classes of nilpotents, applying the nonabelian Hodge correspondence to the associated Slodowy slice defines higher Teichmüller spaces. However, for general nilpotents the geometric significance of the representations associated to Slodowy slices is not clear. In this talk, I will explain why every Variation of Hodge structure in any Slodowy slice in SL(3,C) and SL(4,C) defines an Anosov representation. In rank 3, we will relate such Higgs bundles to complex hyperbolic structures on disc bundles over Riemann surfaces.

High Energy Estimates and the Labourie Conjecture

Peter Smillie

University of Heidelberg

Thursday, April 18, 2024 @ 4:00 PM

High energy harmonic maps to symmetric spaces look almost everywhere like harmonic maps into flats. I will first describe how we used theorems of Mochizuki to that effect in order to find unstable minimal surfaces in locally symmetric spaces of rank at least three, thereby disproving the remaining cases of a conjecture of Labourie. This will then motivate some recent work removing the 'generically regular semisimple' hypothesis from some of Mochizuki's theorems. This is all joint work with Nathaniel Sagman.

Cubic Differentials and Harmonic Maps into a Real Building

JOHN LOFTIN

Rutgers University-Newark

Friday, April 19, 2024 @ 9:00 AM

Higgs bundles and the nonlinear Hodge correspondence provide a way to produce many equivariant harmonic maps from the universal cover of a Riemann surface into symmetric spaces of noncompact type. In general, the construction involves non-explicit solutions to systems of elliptic PDEs (the Hitchin equations) and ODEs (a parallel transport). We are able to say more in an important special limiting case.

Consider a closed Riemann surface S of genus g at least 2 equipped with a holomorphic cubic differential U. Following Hitchin, Labourie and myself, we construct a Higgs bundle from U over S so that the Hitchin system reduces to a single PDE, and the induced harmonic map is a minimal embedding into the symmetric space X=SL(3,R)/SO(3). Along a real ray tU as t goes to infinity, we find an explicit description of the geometry of the limiting minimal surface in the asymptotic cone of X (which is a real building), in terms of the geometry of U.

This is joint work with Andrea Tamburelli and Mike Wolf.

Higgs Bundles and Lie Algebroid Connections

ANDRE OLIVEIRA

University of Porto

Friday, April 19, 2024 @ 10:15 AM

Out of a given holomorphic Lie algebroid L on a compact Riemann surface X, one can consider a corresponding L-connection on a vector bundle over X. By changing the Lie algebroid structure in an appropriate way, this naturally degenerates onto a twisted Higgs bundle on X. This degeneration also fits in Simpson's classical picture using λ -connections. When rk(L) = 1, we use this to study geometric and topological properties of the moduli spaces of L-connections on X, as simple as their dimension or more complicated like their motivic class in the Grothendieck ring of varieties. This is joint work with David Alfaya.

Quantum Matter, Hyperbolic Band Structures, and Moduli Spaces

STEVEN RAYAN

University of Saskatchewan

Friday, April 19, 2024 @ 11:45 AM

The advent of topological materials, a form of physical matter with unusual but useful properties, has brought with it unexpected new connections between physics and pure mathematics. As the name suggests, topology has played a significant role in understanding and classifying these materials. In this talk, I will offer a brief look at a vast extension to this story, arising from my work with various collaborators over the last four years, that sees complex geometry in particular, Riemann surfaces and moduli spaces associated to them, including those of Higgs bundles being used to anticipate new models of quantum matter. Most importantly, calculations may be made regarding the physics of such materials via a hyperbolic approximation of Bloch-Floquet band theory, which has also appeared in our work. There will be lots of pictures.

The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

List of Participants

STEVEN BRADLOW, University of Illinois Urbana-Champaign BRIAN COLLIER, University of California-Riverside PANOS DIMAKIS, Universite du Quebec RON DONAGI, University of Pennsylvania OSCAR GARCIA-PRADA, ICMAT, Madrid BILL GOLDMAN, University of Maryland PETER GOTHEN, University of Porto ANDRIY HAYDYS, Free University of Brussels SIQI HE, Morningside Center, Beijing JOCHEN HEINLOTH, University of Duisberg-Essen NIGEL HITCHIN, University of Oxford JOHANNES HORN, Goethe Universitat Frankfurt DORON LEVY, University of Maryland/Director, Brin MRC JOHN LOFTIN, Rutgers University-Newark **RAFE MAZZEO**, Stanford University SWARNAVA MUKHOPADHYAY, Tata Institute for Fundamental Research MOTOHICO MULASE, University of California-Davis XUESEN NA, University of Illinois Urbana-Champaign ANDY NEITZKE, Yale University ANDRE OLIVEIRA, University of Porto CHARLES OUYANG, Washington University STEVEN RAYAN, University of Saskatchewan KARIM REGA, University of Edinburgh GABRIELE REMBADO, University of Maryland NATHANIEL SAGMAN, University of Luxembourg LAURA SCHAPOSNIK, University of Illinois at Chicago SEBASTIAN SCHULTZ, Johns Hopkins University PETER SMILLIE, University of Heidelberg HARTMUT WEISS, University of Kiel RICHARD WENTWORTH, University of Maryland MIKE WOLF, Georgia Tech University