# BRIN MATHEMATICS RESEARCH CENTER

Geometric Approaches to the Local Langlands Program

March 10-13, 2025

About the workshop

This workshop aims to explore connections between p-adic Arthur and ABV packets and geometric representation theory/the geometric Langlands program. It will feature talks from experts in both areas. The primary goal of this workshop is to foster new research directions and collaborations.
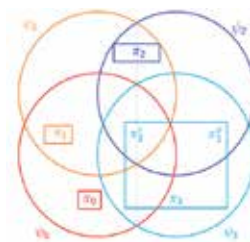
Organizers

Alexander Bertoloni Meli, Bonn & Boston Universities

Clifton Cunningham, University of Calgary

Peter Dillery, University of Maryland

Thomas Haines University of Maryland

Participants

Jeff Adams, University of Maryland

Kristaps Balodis, University of Calgary

Ana Caraiani, Imperial College London

Jose Cruz, University of Calgary

Gurbir Dhillon, Yale University

Arnaud Eteve, MPIM, Bonn

Linus Hamann, Stanford & Harvard Universities

David Hansen, National University of Singapore

Alexander Hazeltine, University of Michigan

Eugen Hellmann, University of Munster

David Helm, Imperial College London

Teruhisa Koshikawa, Kyoto University

Baiying Liu, Purdue University

Siddharth Mahendraker, Boston College

Lucas Mason Brown, University of Texas at Austin

David Nadler, University of California, Berkeley

Kieu Hieu Nguyen, UVSQ

Sam Raskin, Yale University

Mishty Ray, University of Calgary

Nick Rozenblyum, University of Toronto

James Steele, University of Calgary

Yakov Varshavsky, Hebrew University of Jerusalem

David Vogan, Massachusetts Institute of Technology

Bin Xu, Tsinghua University

Zecheng Yi, Boston University

Mingjia Zhang, Princeton University

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DEPARTMENT OF MATHEMATICS

Contents

Schedule at a Glance

	Monday	Tuesday	Wednesday	Thursday
8:00				
			Breakfast	
9:00	Breakfast		Hansen	Breakfast
	Cunningham	Breakfast		Steele
10:00		Ray	Coffee Break	
	Coffee Break	Coffee Break	Xu	Coffee Break
11:00	Helm	Raskin		Dhillon
			Lunch	
12:00	Lunch			
		Lunch	Rozenblyum	
13:00				
	Koshikawa	Liu	Group Activity	
14:00				
	Coffee Break	Coffee Break		
15:00	Zhang	Hellmann		
16:00	Coffee Break	Coffee Break		
	Hamann	Q and A Session		
17:00				
18:00				

Workshop Overview

This is a 4-day workshop that aims to explore connections between p-adic Arthur and ABV packets and geometric representation theory/the geometric Langlands program. It will feature talks from experts in both areas. The primary goal of this workshop is to foster new research directions and collaborations.

Organizing committee

CLIFTON CUNNINGHAM University of Calgary

PETER DILLERY University of Maryland

THOMAS HAINES, University of Maryland

ALEXANDER BERTOLONI MELI, Bonn University / Boston University

Workshop Schedule

Monday, March 10, 2025

8:50 - 9:20	Breakfast
9:20 - 9:30	Doron Levy (University of Maryland/Director, Brin MRC) *Opening*
9:30 - 10:30	Clifton Cunningham (University of Calgary) *New Results and Open Problems on ABV-Packets for P-Adic Groups*
10:30 - 11:00	Coffee Break
11:00 - 12:00	David Helm (Imperial College London) *Co-Whittaker Families for Quasi-Split Groups*
12:00 - 1:30	Lunch
1:30 - 2:30	Teruhisa Koshikawa (Kyoto University) *A-Parameters, Eigensheaves, and Cuspidal Coherent Sheaves*
2:30 - 3:00	Coffee Break
3:00 - 4:00	Mingjia Zhang (Princeton University) *Intersection Cohomology of Shimura Varieties*
4:00 - 4:30	Coffee Break
4:30 - 5:30	Linus Hamann (Stanford University & Harvard University) *Geometric Eisenstein Series and Intertwining Operators*

Tuesday, March 11, 2025

9:00 - 9:30   Breakfast

9:30 - 10:30   Mishty Ray (University of Calgary)
Geometric Analogues to Local Arthur Packets for P-Adic Classical Groups

10:30 - 11:00   Coffee Break

11:00 - 12:00   Sam Raskin (Yale University)
Arthur Parameters Over Function Fields

12:00 - 12:15   Group Photo

12:15 - 1:30   Lunch

1:30 - 2:30   Baiying Liu (Purdue University)
On Representations of Arthur Type and Unitary Dual for P-Adic Classical Groups

2:30 - 3:00   Coffee Break

3:00 - 4:00   Eugen Hellmann (University of Munster)
Sen-Theory via Moduli Spaces

4:00 - 4:30   Coffee Break

4:30 - 5:30   Q and A Session

Wednesday, March 12, 2025

8:30 - 9:00   Breakfast

9:00 - 10:00   David Hansen (National University of Singapore)
                 Functoriality Functors

10:00 - 10:30   Coffee Break

10:30 - 11:30   Bin Xu (Tsinghua University)
                 On the Structure of Arthur Packets for Real Unitary Groups

11:30 - 12:30   Lunch

12:30 - 1:30   Nick Rozenblyum (University of Toronto)
                 Higher Characters in Representation Theory

1:30 - 5:30   Group Activity

5:45 - 8:30   Conference Dinner

THURSDAY, MARCH 13, 2025

9:00 - 9:30   BREAKFAST

9:30 - 10:30   JAMES STEELE (University of Calgary)
Koszul Duality of Generalised Steinberg Representations in the Local Langlands Correspondence for P-Adic Groups

10:30 - 11:00   COFFEE BREAK

11:00 - 12:00   GURBIR DHILLON (Yale University)
On Local Geometric Langlands in Depth Zero

12:00 - 12:10   WORKSHOP CLOSING

Abstracts of talks

New Results and Open Problems on ABV-Packets for P-Adic Groups

CLIFTON CUNNINGHAM

University of Calgary

Monday, March 10, 2025 @ 9:30 AM

This talk will recall known properties of ABV-packets for p-adic groups, review the constructions needed to define and calculate them, and present new results. We will also discuss open problems, especially as related to various perspectives on a categorical Langlands correspondence.

Co-Whittaker Families for Quasi-Split Groups

David Helm

Imperial College London

Monday, March 10, 2025 @ 11:00 AM

In his tensor factorization of the completed cohomology of the modular tower (with Z_p-coefficients), Emerton observed that the local factors corresponding to primes ℓ different from p could be uniquely characterized, up to isomorphism, by a short list of properties relating such a factor to the associated local Galois representation (in particular, such factors satisfied a form of Ihara's lemma and agreed generically with the representations of GL_2 associated to the local Galois representation via local Langlands.) In later joint work Emerton and I proved that, given an n-dimensional local Galois representation, there was at most one family of representations of GL_n over the same base that satisfied a similar set of desiderata; such families were later shown to always exist by joint work of mine with Gil Moss. We show that an analogous construction can be made for any quasi-split group over a local field F, and explain a (conjectural) connection between this construction and the categorical local Langlands correspondence. This is joint work with Jean-Francois Dat, Rob Kurinczuk, and Gil Moss.

A-Parameters, Eigensheaves, and Cuspidal Coherent Sheaves

Teruhisa Koshikawa

Kyoto University

Monday, March 10, 2025 @ 1:30 PM

I will talk about my project with Bertoloni Meli, where we work entirely on the spectral/Galois side of the categorical local Langlands. We construct eigensheaves in the sheared sense for any A-parameters. To verify their expected properties further, we study the structure of the spectral side and introduce the notion of cuspidal coherent sheaves. A large part of this project is inspired by the comparison with the ABV picture, and I hope to explain that.

Intersection Cohomology of Shimura Varieties

Mingjia Zhang

Princeton University

Monday, March 10, 2025 @ 3:00 PM

The cohomology of Shimura varieties provides a geometric setup for the Langlands correspondence over number fields. For noncompact Shimura varieties, compared to the usual or compact support cohomology, the intersection cohomology has a cleaner relation to automorphic forms through its relation to L2 cohomology. We discuss results on the intersection cohomology of Shimura varieties in the context of the Fargues-Scholze local Langlands correspondence. This is based on joint work in progress with Ana Caraiani and Linus Hamann.

Geometric Eisenstein Series and Intertwining Operators

Linus Hamann

Stanford University & Harvard University

Monday, March 10, 2025 @ 4:30 PM

Let G be a split connected reductive group over the p-adic numbers with a parabolic $P = MU$. Let π_M be a generic supercuspidal representation with L-parameter ϕ_M. We write ϕ for the composition of ϕ_M with the map $\hat{M} \longrightarrow \hat{G}$. In the classical theory of automorphic forms, the Langlands-Shahidi method relates the intertwining operators of the induction of π_M from P to G to the pole of the local L-function attached to ϕ_M and r_{ad}, the adjoint representation of M on the Lie algebra of U at s = 1. In this talk, we discuss a parallel story in the geometrization of the local Langlands correspondence. In particular, to such a parameter ϕ_M, we discuss the existence of an eigensheaf S_ϕ on Bun_G, the moduli stack of G-bundles on the Fargues-Fontaine curve constructed using a geometric analogue of Eisenstein series. This eigensheaf takes a remarkably simple form under the assumption that the Galois cohomology of the representation $r_{ad} \circ \phi_M$ and $r_{ad} \circ \phi_M^\vee$ vanish, which is equivalent to saying that the local adjoint L-function does not have a pole at s = 1. By combining the simple structure of S_ϕ in this case with the eigensheaf property , one is able to actually show that the intertwining operators of the induction of π_M are an isomorphism, in a way that recovers the work of Langlands-Shahidi but using purely local geometry instead of global automorphic methods.

Geometric Analogues to Local Arthur Packets for P-Adic Classical Groups

Mishty Ray

University of Calgary

Tuesday, March 11, 2025 @ 9:30 AM

ABV-packets are proposed generalizations of local Arthur packets; they arise out of Vogan's geometric perspective on the local Langlands correspondence for p-adic groups. In particular, this proposal says an ABV-packet for a Langlands parameter of Arthur type coincides with the corresponding local Arthur packet. This is stated as Vogan's conjecture in the nonarchimedean adaptation of the work of ABV, by Cunningham et al. This talk aims to report on the progress of Vogan's conjecture. In joint work with Cunningham, we prove this conjecture for p-adic general linear groups. The proof of the conjecture is in progress for p-adic classical groups in a joint project with Cunningham, Hazeltine, Liu, Lo, and Xu. In this project, we generalize a key technique we used for general linear groups. In this talk, we will set up Vogan's conjecture and discuss this interesting aspect of the proof.

Arthur Parameters Over Function Fields

Sam Raskin

Yale University

Tuesday, March 11, 2025 @ 11:00 AM

In the setting of function fields, Zhu has presented a "Fourier-transform" style version of the Langlands conjectures for function fields, mirroring what happens for geometric Langlands.
In joint work with Arinkin-Gaitsgory-Kazhdan-Rozenblyum-Varshavsky, we showed how this conjecture follows from a version of the geometric Langlands conjecture for l-adic sheaves. I will review these developments and then discuss how (at least conjecturally) Arthur parameters can be incorporated into the story.

On Representations of Arthur Type and Unitary Dual for P-Adic Classical Groups

BAIYING LIU

Purdue University

Tuesday, March 11, 2025 @ 1:30 PM

In this talk, towards the unitary dual problem for p-adic classical groups, I will introduce a refinement of a conjecture of Marko Tadic: A representation of good parity is unitary if and only if it is of Arthur type; and a conjectural description of the whole unitary dual. We verified them for representations of corank at most 3 for symplectic or split odd special orthogonal groups, based on Tadic's classification. To this end, we developed new algorithms to determine whether a given irreducible representation is of Arthur type and gave an inductive approach to classify the family of unitary representations that are of Arthur type. As applications, we carried out the explicit list on representations of Arthur type with corank at most 3 and classified several new families of representations of Arthur type. This is a joint work with Alexander Hazeltine, Dihua Jiang, Chi-Heng Lo, and Qing Zhang.

Sen-Theory via Moduli Spaces

EUGEN HELLMANN

University of Munster

Tuesday, March 11, 2025 @ 3:00 PM

I will report on joint work with Ben Heuer on a formulation of Sen-theory in terms of moduli spaces that parallels results in the context of the p-adic Simpson correspondence: For a finite extensions F of Q_p we study moduli stacks, on the category of rigid analytic representations, of semi-linear Gal_F representations on C_p-vector spaces respectively on projective B_dR^+-modules. These stacks admit a morphism (via the theory of Hodge-Tate-Sen weights) to an affine space and become (étale locally over this base) isomorphic to stacks defined in terms of linear algebra. This construction also applies to reductive groups over F (more precisely to their L-groups) instead of GL_n.

Functoriality Functors

David Hansen

National University of Singapore

Wednesday, March 12, 2025 @ 9:00 AM

Let G, H be a pair of quasisplit reductive groups over a nonarchimedean local field E, with fixed choices of Whittaker data. Given any L-homomorphism h: $^LG \longrightarrow {}^LH$, I will explain how to construct a canonical Langlands functoriality functor $scrL_h$: $D(Bun_G) \to D(Bun_H)$. Although this construction is guided by considerations in categorical local Langlands, it is totally unconditional. I'll formulate some conjectures about these functors and explain what can be proved.

On the Structure of Arthur Packets for Real Unitary Groups

Bin Xu

Tsinghua University

Wednesday, March 12, 2025 @ 10:30 AM

Moeglin and Renard gave an explicit construction of Arthur packets for real unitary groups by cohomological induction and parabolic induction. The cohomological induction considered in their case is in the weakly fair range and is very difficult to compute. Nevertheless, Trapa had given a combinatorial solution to this computation problem long ago by studying the annihilators and associated varieties of the cohomological induced representations. In this talk, we would like to propose a new solution to this problem inspired by the structure of Arthur packets for p-adic classical groups. We will also discuss a proof through the geometry of Langlands parameters space. This is joint work with Taiwang Deng, Chang Huang and Qixian Zhao.

Higher Characters in Representation Theory

Nick Rozenblyum

University of Toronto

Wednesday, March 12, 2025 @ 12:30 PM

I will describe a generalization of character theory for categorical representations of groups in the l-adic setting. In this case of finite groups of Lie type, this provides a natural framework for Deligne-Lusztig theory. This theory also plays a key role in establishing a relationship between the geometric and classical Langlands correspondences. This is based on joint work with Gaitsgory and Varshavsky.

Koszul Duality of Generalised Steinberg Representations in the Local Langlands Correspondence for P-Adic Groups

James Steele

University of Calgary

Thursday, March 13, 2025 @ 9:30 AM

In this talk, we discuss extensions between smooth, irreducible representations of p-adic groups and study their interactions with A-packets and unitary representations. In particular, we show that the category of equivariant perverse sheaves on the moduli space of Langlands parameters that classify generalised Steinberg representations is equivalent to the category of modules over the Yoneda algebra generated by these representations.

On Local Geometric Langlands in Depth Zero

Gurbir Dhillon

Yale University

Thursday, March 13, 2025 @ 11:00 AM

A fundamental theorem of Bezrukavnikov establishes local geometric Langlands with tame ramification and unipotent monodromy. We will review this result for non-specialists, and then discuss some joint works with Li—Yun–Zhu and Taylor, which extend this to the full equivalence in depth zero for restricted variation and Betti families, as conjectured by Bezrukavnikov and Ben-Zvi–Nadler, respectively. Time permitting, we will describe some ongoing work with Yang on the story in positive depth.

The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

List of Participants

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Kristaps Balodis, University of Calgary
Alexander Bertoloni Meli, Boston University
Ana Caraiani, Imperial College London
Jose Cruz, University of Calgary
Clifton Cunningham, University of Calgary
Gurbir Dhillon, Yale University
Peter Dillery, University of Maryland
Arnaud Eteve, Max Planck Institute for Mathematics in Bonn
Ian Gleason, University of Bonn
Thomas Haines, University of Maryland
Linus Hamann, Stanford University & Harvard University
David Hansen, National University of Singapore
Alexander Hazeltine, University of Michigan
Eugen Hellmann, University of Munster
David Helm, Imperial College London
Teruhisa Koshikawa, Kyoto University
Doron Levy, University of Maryland/Director, Brin MRC
Baiying Liu, Purdue University
Joao Lourenço, University of Munster
Siddharth Mahendraker, Boston College
Lucas Mason Brown, University of Texas at Austin
David Nadler, University of California, Berkeley
Kieu Hieu Nguyen, University of Versailles Saint-Quentin
Sam Raskin, Yale University
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Bin Xu, Tsinghua University
Zecheng Yi, Boston University
Jize Yu, Rice University
Mingjia Zhang, Princeton University